

Trial and Improvement 1 - PDF Copy

The presentation contains the slides below with the objective of showing how to: **Use trial and improvement to find approximate solutions for equations where a solution cannot be found by simple algebraic method.** There are animated explanations and problems with answers,

Trial and Improvement

Objective: Use trial and improvement to find approximate solutions for equations where a solution cannot be found by simple algebraic method

Grade D - A Topic



1

What is trial and improvement?

Trial and improvement is a way of solving equations by trying a number and then using the results to improve on it.

Suppose, for example, we wanted to find the value of $x = \sqrt{12}$

To do this, we would use the button on a calculator

But if this was broken, we could use trial and improvement like this...



2

Try $x = 3$

$x = \sqrt{12}$
 $\rightarrow x^2 = 12$

Try	Workout	Comment
3	$3^2 = 9$	Too small
4	$4^2 = 16$	Too big

Answer between 3 and 4

3 is too small so try 4

We know that the answer must be between 3 and 4



3

Try $x = 3.5$

$x = \sqrt{12}$
 $\rightarrow x^2 = 12$

Try	Workout	Comment
3	$3^2 = 9$	Too small
4	$4^2 = 16$	Too big
3.5	$3.5^2 = 12.25$	Too big
3.4	$3.4^2 = 11.56$	Too small

Answer between 3 and 4

3.5 is too big so try 3.4

We know that the answer must be between 3 and 4

Try the mid-point $x = 3.5$



4

Try $x = 3.45$

$x = \sqrt{12}$
 $\rightarrow x^2 = 12$

Try	Workout	Comment
3	$3^2 = 9$	Too small
4	$4^2 = 16$	Too big
3.5	$3.5^2 = 12.25$	Too big
3.4	$3.4^2 = 11.56$	Too small
3.45	$3.45^2 = 11.9025$	Too small
3.46	$3.46^2 = 12.0116$	Too big

Answer between 3.4 and 3.5

Try the mid-point $x = 3.45$



5

$x = \sqrt{12}$
 $\rightarrow x^2 = 12$

Try	Workout	Comment
3.46	$3.46^2 = 11.9716$	Too small
3.47	$3.47^2 = 12.0409$	Too big

To 2 dp the answer is 3.46 or 3.47

To decide which, try the mid-point 3.465

3.465 $3.465^2 = 12.006225$ Too big

Because it's too big, the answer to 2 dp is 3.46

$x = 3.46$



6

More difficult equations can be solved by trial and improvement. Here is an example...



7

Here is a quadratic equation $x^2 - x = 5$

As you may know it is possible to solve it using algebra and it has two answers

But as an example, we show how it can be solved using trial and improvement...



8

Try $x = 3$

$x^2 - x = 5$

$3^2 - 3 = 6$

Answer of 6

This is ~~too big~~. The answer is smaller than 3

So try x is 2...



9

Try $x = 2$

$x^2 - x = 5$

$2^2 - 2 = 2$

Answer of 2

Now, the answer is too small

Because 3 is ~~too big~~ and 2 is ~~too small~~ we know that the answer must be between 2 and 3



10

Find a value of x for $x^2 - x = 5$

Try	Workout	Comment
3	$3^2 - 3 = 6$	Too big
2	$2^2 - 2 = 2$	Too small

Answer between 2 and 3

It is helpful to put the results into a table like this...



11

Find a value of x for $x^2 - x = 5$

Try	Workout	Comment
3	$3^2 - 3 = 6$	Too big
2	$2^2 - 2 = 2$	Too small
2.5	$2.5^2 - 2.5 = 3.75$	Too small
2.6	$2.6^2 - 2.6 = 4.16$	Too small
2.7	$2.7^2 - 2.7 = 4.59$	Too small
2.8	$2.8^2 - 2.8 = 5.04$	Too big

Answer between 2.7 and 2.8

We know that the answer is between 2.7 and 2.8 so try 2.75

by a value that is too big



12

Find a value of x for $x^2 - x = 5$

Try	Workout	Comment
3	$3^2 - 3 = 6$	Too big
2	$2^2 - 2 = 2$	Too small
2.5	$2.5^2 - 2.5 = 3.75$	Too small
2.6	$2.6^2 - 2.6 = 4.16$	Too small
2.7	$2.7^2 - 2.7 = 4.59$	Too small
2.8	$2.8^2 - 2.8 = 5.04$	Too big

Answer between 2.7 and 2.8

Now, we know that the answer is between 2.7 and 2.8

Try 2.75



13

Find a value of x for $x^2 - x = 5$

Try	Workout	Comment
2.75	$2.75^2 - 2.75 = 4.8125$	Too small
2.76	$2.76^2 - 2.76 = 4.8576$	Too small
2.77	$2.77^2 - 2.77 = 4.9029$	Too small
2.78	$2.78^2 - 2.78 = 4.9488$	Too small
2.79	$2.79^2 - 2.79 = 4.9941$	Too small
2.80	$2.80^2 - 2.80 = 5.04$	Too big

Answer between 2.79 and 2.80

We are getting very close. If we want an answer to 2 dp, we would try 2.795...



14

Find a value of x for $x^2 - x = 5$

Try	Workout	Comment
2.79	$2.79^2 - 2.79 = 4.9941$	Too small
2.80	$2.80^2 - 2.80 = 5.04$	Too big
2.795	$2.795^2 - 2.795 = 5.017025$	Too big

Answer between 2.79 and 2.80

Because 2.795 is too big, the answer is 2.79 to 2 dp

If 2.795 had been too small, the answer would have been 2.80 to 2 dp



15

Continued...

Find a value of x for $x^2 - x = 5$

Try	Workout	Comment
2.79	$2.79^2 - 2.79 = 4.9941$	Too small
2.80	$2.80^2 - 2.80 = 5.04$	Too big

Answer between 2.79 and 2.80

2.795	$2.795^2 - 2.795 = 5.017025$	Too big
-------	------------------------------	---------

By using trial and improvement, we have found an answer

$x^2 - x = 5$
 $x = 2.79$ to 2 dp

In exam situations, a starting point and a table are usually provided like this...

There is a value of x between 4 and 5 that solves the equation $x^2 - x = 16$, use trial and improvement to find this value to 2 dp.

Try	Workout	Comment
4	$4^2 - 4 = 12$	Too small
5	$5^2 - 5 = 20$	Too big

There is an answer between 4.5 and 4.6

4.5	$4.5^2 - 4.5 = 15.75$	Too small
4.6	$4.6^2 - 4.6 = 16.56$	Too big
4.55	$4.55^2 - 4.55 = 16.1525$	Too small
4.55	$4.55^2 - 4.55 = 16.2336$	Too big

Try 4.55

To 2 dp the answer is 4.55 or 4.56

If we are told that there is a value of x between 4 and 5 that solves the equation $x^2 - x = 16$, use trial and improvement to find this value to 2 dp.

$x = 4.55$

To decide which is the correct answer to 2 dp, try 4.555

Because it's too big, try 4.55

To 2 dp the answer is 4.55 or 4.56

Try	Workout	Comment
4.55	$4.55^2 - 4.55 = 16.1525$	Too small
4.56	$4.56^2 - 4.56 = 16.2336$	Too big
4.555	$4.555^2 - 4.555 = 16.193025$	Too big

More practice...

- The equation $x^2 - x = 14$ has a solution between 4 and 5. Use trial and improvement to find its value to 2 dp. $x = 4.27$
- The equation $x^2 - 3x = 12$ has a solution between 4 and 5. Use trial and improvement to find its value to 2 dp. $x = 5.27$
- The equation $3x^2 - 2x = 20$ has a solution between 4 and 5. Use trial and improvement to find its value to 2 dp. $x = 2.94$

☆ 16

☆ 17

☆ 18

☆ 19

☆ 20

Cubic Equations

Equations like $x^3 + x = 21$ are called cubic equations and probably the easiest method of finding their solutions is by trial and improvement. Here is an example of a typical test type question...

There is a solution to $x^3 + x = 21$ between 2 and 3. Use trial and improvement to find this to 2 dp.

Try	Workout	Comment
2	$2^3 + 2 = 10$	Too small
3	$3^3 + 3 = 30$	Too big

Solution between 2 and 3

$2.5^3 + 2.5 = 18.125$

Try mid-point 2.5

There is a solution to $x^3 + x = 21$ between 2 and 3. Use trial and improvement to find this to 2 dp.

Try	Workout	Comment
2	$2^3 + 2 = 10$	Too small
3	$3^3 + 3 = 30$	Too big
2.5	$2.5^3 + 2.5 = 18.125$	Too small
2.6	$2.6^3 + 2.6 = 20.176$	Too small
2.7	$2.7^3 + 2.7 = 22.243$	Too big

Solution between 2.6 and 2.7

To 2 dp the answer is 2.63 or 2.64

Try mid-point 2.65

There is a solution to $x^3 + x = 21$ between 2 and 3. Use trial and improvement to find this to 2 dp.

2.64	$2.64^3 + 2.64 = 20.82$ to 2 dp	Too small
2.63	$2.63^3 + 2.63 = 20.63$ to 2 dp	Too small
2.635	$2.635^3 + 2.635 = 20.93$ to 2 dp	Too small

To 2 dp the answer is 2.63 or 2.64

To decide which try 2.635

If it's too big $x = 2.63$

If it's too small $x = 2.64$

$x = 2.64$ to 2 dp

Using the given starting values, solve these cubic equations giving your answer to 2 dp

- $x^3 + x = 40$ (A solution between 3 and 4) $x = 3.22$
- $x^3 + x = 100$ (A solution between 4 and 5) $x = 4.57$
- $x^3 - x = 80$ (A solution between 4 and 5) $x = 4.39$
- $x^3 + 2x = 15$ (A solution between 2 and 3) $x = 2.20$

☆ 21

☆ 22

☆ 23

☆ 24

☆ 25

Exponential Equations

Equations where a number is raised to some power of x such as $2^x = 20$ for example are called exponential equations.

Solutions to these can also be found by trial and improvement. Here is a typical example...

There is a solution to $2^x = 20$ between 4 and 5. Use trial and improvement to find this to 2 dp.

Try	Workout	Comment
4	$2^4 = 16$	Too small
5	$2^5 = 32$	Too big
4.5	$2^{4.5} = 22.63$ to 2 dp	Too big

Solution between 4 and 5

Try mid-point 4.5

There is a solution to $2^x = 20$ between 4 and 5. Use trial and improvement to find this to 2 dp.

Try	Workout	Comment
4	$2^4 = 16$	Too small
5	$2^5 = 32$	Too big
4.5	$2^{4.5} = 22.63$ to 2 dp	Too big
4.4	$2^{4.4} = 21.11$ to 2 dp	Too big
4.3	$2^{4.3} = 19.70$ to 2 dp	Too small

Solution between 4.3 and 4.4

There is a solution to $2^x = 20$ between 4 and 5. Use trial and improvement to find this to 2 dp.

Try	Workout	Comment
4	$2^4 = 16$	Too small
5	$2^5 = 32$	Too big
4.5	$2^{4.5} = 22.63$ to 2 dp	Too big
4.4	$2^{4.4} = 21.11$ to 2 dp	Too big
4.3	$2^{4.3} = 19.70$ to 2 dp	Too small
4.35	$2^{4.35} = 20.11$ to 2 dp	Too small
4.34	$2^{4.34} = 20.04$ to 2 dp	Too small
4.33	$2^{4.33} = 20.11$ to 2 dp	Too big
4.32	$2^{4.32} = 19.97$ to 2 dp	Too small

Try mid-point 4.35

To 2 dp the solution is 4.32 or 4.33

To decide which, try 4.325

If it's too big $x = 4.32$

If it's too small $x = 4.33$

$x = 4.32$ to 2 dp

There is a solution to $2^x = 20$ between 4 and 5. Use trial and improvement to find this to 2 dp.

4.33	$2^{4.33} = 20.11$ to 2 dp	Too big
4.32	$2^{4.32} = 19.97$ to 2 dp	Too small
4.325	$2^{4.325} = 20.04$ to 2 dp	Too big

To 2 dp the solution is 4.32 or 4.33

To decide which, try 4.325

If it's too big $x = 4.32$

If it's too small $x = 4.33$

$x = 4.32$ to 2 dp

☆ 26

☆ 27

☆ 28

☆ 29

☆ 30

Using the given starting values, solve these exponential equations giving your answer to 2 dp

- $3^x = 40$ (A solution between 3 and 4) $x = 3.36$
- $7^x = 90$ (A solution between 2 and 3) $x = 2.31$
- $3^x = 150$ (A solution between 4 and 5) $x = 4.56$
- $5^x = 250$ (A solution between 3 and 4) $x = 3.43$

☆ 31